

Reaching Paris Agreement Goal through CDR/DAC Development: a Compact OR Model

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Abstract

A compact operations research (OR) model is proposed to analyse the prospects of meeting the Paris Agreement targets when direct air capture technologies can be used or not. The main features of the model are (i) the representation of the economy and energy use with a nested constant elasticity of substitution production function; (ii) the representation of climate policy through the use of a safety emissions budget concept; and (iii) the representation of an international emissions trading scheme for the implementation of climate policy. Using dynamic optimisation, several contrasting scenarios are analysed and the potential use of the model in future developments of climate/economy modelling is discussed.

Keywords: Climate policy, Optimal economic growth, Dynamic optimisation model, Market equilibrium constraints, CO₂ direct reduction.

1. Introduction

It is now well established that, in order to achieve the objectives of the Paris Agreement, a regime of zero net emissions must be reached by 2050 or 2070 at the latest. Direct CO₂ emission reduction technologies, in particular BECCS¹

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¹Biomass Energy with Carbon Capture and Sequestration.

5 and DAC/CDR² technologies, will then play an important role by generating negative emissions to offset the GHGs emitted by fossil fuel technologies³. The objective of this article is to propose a compact OR model that can provide an economic assessment of the possible contribution of CDR/DAC technologies to the achievement of the Paris Climate Agreement objectives.

10 The model is based on a Ramsey-style optimal economic growth paradigm, with the economy, energy and negative emissions production described by CES functions [1]. The originality of the approach lies in the representation of useful and secondary energy production as well as the production of negative emissions from CDR/DAC, through the use of capital (plants), labour and primary
15 fossil energy, which is the source of GHG emissions. This OR model provides a framework to explore the macroeconomic costs of climate policies to achieve the Paris Agreement targets when CDR/DAC technology can and cannot be used. By providing an overview of the more global optimal economic growth, this approach complements the work done in [2], concerning the possible role of
20 CDR/DAC development in the climate policy of oil and gas producing countries and [3], where an oligopoly game of CDR technology development in a steady-state zero net emissions climate regime is proposed. CDR/DAC technologies have been evaluated in IAMs⁴ that include an optimal economic growth paradigm “a la Ramsey” [4] and [5], but these models use a different description
25 of the economic good and the energy production processes. The compact OR model presented here can also be linked to the stochastic control model [6] and differential game model [7, 8] already proposed to analyse global climate policy. As in these previous works, we represent climate policy through the sharing of a remaining safety emissions budget (SEB), as suggested in [9, 10]. In addition,
30 a representation of an international emissions trading system is included in this OR model. The global supply of emission rights (permits) will determine the

²Direct Air Capture and Carbon Dioxide Removal.

³See, for example, The Economist’s briefing or Shell Corp’s Sky scenario where BECCS is massively used to produce negative emissions.

⁴Integrated assessment models.

price of the permit and emission levels in each region will be such that the marginal abatement cost is equal to the price. Finally, to calibrate the DAC production function, we use the techno-economic analyses of [11] and [12].

35 In the rest of the paper, we describe in Section 2 the main features of the model and we present in Section 3 some scenarios produced using the model with different optimisation criteria and climate policy constraints. In Section 4, we discuss the results already obtained and conclude.

2. The model

40 2.1. Economic structure

We regroup the world countries in three “coalitions” j called BRIC⁵, OECD and ROW⁶, respectively. They represent groups of nations in similar states of development. In each group, we represent an economy where a general economic good is produced with three inputs: labour L_0 , capital K_0 and energy E_0 . The energy input E_0 , or useful energy, can be obtained from two kinds of inputs, 45 one fossil, noted enf_0 , and the other renewable, enr_0 . The fossil energy input is obtained from two factors, respectively the fossil fuel power plants K_1 and the fossil primary energy enp_1 . The renewable input is produced by zero-emission plants, represented by the capital K_2 . GHG emissions are associated with the fossil energy primary source enp_1 . Negative emissions v can be produced by 50 CDR/DAC technologies using three production factors, labor L_3 , capital K_3 and useful energy E_3 . The energy mix, fossil vs. renewable, is supposed to be common to energy input for the general productive economy, E_0 and the DAC/CDR sector E_3 .

55 This production structure, schematised in Figure 3 in Appendix will be mathematically described in nested CES function that are calibrated from GTAP 10 database for the reference year 2014 [13]. All economic variables are expressed in US\$₂₀₁₄ using market exchange rates. Energy consumptions, from fossil or

⁵Brazil, Russia, India, China.

⁶Rest of the world.

renewable sources are expressed in physical terms (peta-joule, PJ); for calibration purpose they are obtained from the energy balances published online by the International Energy Agency (IEA)⁷. The CO₂ emissions from fuel combustion are also obtained from the IEA [14]. The DAC technology is calibrated using [12]. We use the cost estimates for a DAC system using the sodium/calcium hydroxide option (see Table 2.5 in [12]). We retain the total cost of US\$ 430 per ton of CO₂ captured.

Population levels from 2014 to 2100, expressed in million of people in Table 11, are based on the World Population Prospects 2019 done by the United Division [15]. We use the medium variant scenario. For the whole world, it varies from $7'295 \times 10^6$ (7.295 billion) people in 2014 up to $10'875 \times 10^6$ (10.875 billion) people in 2100. After 2100 we assume a steady state for population in different regions.

2.2. Dynamics

We consider a time set $t \in \{0, 1, \dots, T\}$, where each period corresponds to a number of years Ny . In this application, we take decadal periods ($Ny = 10$). The dynamic model has five state variables, which are the capital stocks $K_i(t, j); i = 0, 1, 2, 3$, and the remaining emission budget $b(t, j)$, for coalition j at period t , and five control variables, which are the investment levels $I_i(t, j); i = 0, 1, 2, 3$, and the supply $\omega(t, j)$ of emission permits by coalition j at period t . The state equations for capital are given in (1)-(2). In (2) the parameter $IB_3(t, j)$ is an upper bound for investment in DAC technology that limits the availability of this technology over time.

$$K_i(t, j) = K_i(t - 1, j)(1 - \mu_j) + Ny \cdot I_i(t - 1, j), \quad (1)$$

$$K_i(j, 0) = K_i^0(j), \quad i = 0, 1, 2, 3, \quad \forall t, \quad \forall j,$$

$$I_3(t, j) \leq IB_3(t, j), \quad (2)$$

⁷<https://www.iea.org/reports/world-energy-balances-overview>

The cumulative emissions budget, since the beginning of the industrial revolution, compatible with a 60% probability of limiting the temperature increase below 1.5 °C has been evaluated at 1 trillion tons of carbon [16]. From this figure we evaluate the remaining Safety Emissions Budget (SEB) at $B = 1'170$ Gt CO₂⁸. The remaining SEB $b(t, j)$ for each coalition of countries will decrease by the amount of emissions permits $Ny \cdot \omega_i(t - 1, j)$ supplied by the coalition, if there is a carbon market or, more directly by the emissions level $Ny \cdot em(t - 1, j)$ of the coalition. The remaining SEB will be replenished by the amount of negative emissions $Ny \cdot v(t - 1, j)$. The parameter $\theta_j \in [0, 1]$ is the share of the SEB given to the coalition j ; one must have $\sum_j \theta_j = 1$. The parameters θ_j , $j = BRIC, OECD, ROW$, summarise in this model the climate negotiations. In summary the SEB dynamics is, for all coalitions j

$$b(t, j) = b(t - 1, j) - Ny \cdot \omega_i(t - 1, j) + Ny \cdot v(t, j) \quad t = 1 \dots T, \quad (3)$$

$$b(0, j) = \theta_j B, \quad (4)$$

if a carbon market market exists, or

$$b(t, j) = b(t - 1, j) - Ny \cdot em(t - 1, j) + Ny \cdot v(t, j), \quad t = 1 \dots T, \quad (5)$$

$$b(0, j) = \theta_j B, \quad (6)$$

$$\sum_j b(t, j) \geq 0, \quad t = 1 \dots T, \quad (7)$$

if there is no market. By imposing a global remaining SEB that remains non-negative (Eq. 7), we impose a climate constraint with no overshooting. We prohibit overshooting for each coalition if we impose $b(t, j) \geq 0, \forall t$.

2.3. Criteria

The periodic discount factor is given by $\beta(t) = 1/(1 + r)^{Ny \cdot t}$, with $r = 3\%$. It is used, in the performance criterion $\Phi = \sum_j \phi(j)$, which is maximised under the constraints of the dynamic model to obtain the desired scenarios. For each

⁸Recall that 3.66 t CO₂ correspond to 1tC.

coalition j the expression $\phi(j)$ represents the discounted sum of utility derived from consumption for its population.

$$\phi(j) = \sum_{t=0}^{T-1} \beta(t)PV \cdot L(t, j) \log(C(t, j)/L(t, j)), \quad j = \text{BRIC, OECD, ROW}, \quad (8)$$

where $PV = \sum_{s=1}^{Ny} (1+r)^{(1-s)}$ is the present value factor at each time t . In (8) $\log(C(t, j)/L(t, j))$ represents the utility derived from per-capita consumption; $C(t, j)$ is the consumption level by coalition j at period t , given by

$$C(t, j) = Y(t, j) - \sum_{i=0,1,2,3} I_i(t, j) - \pi(t, j)enp_1(t, j), \quad (9)$$

where $\pi(t, j)$ is the price of primary fossil energy.

To compare different scenarios we shall use another welfare criterion $W(j)$ for each coalition j . It corresponds to the discounted sum of per-capita consumption, net of the revenue from permit trading, over the whole horizon 2020-2160. For coalition j , we have

$$W(j) = \sum_{t=0}^{T-1} \beta(t)PV \frac{C(t, j) + p(t)(\omega(t, j) - emf(t, j))}{L(t, j)}, \quad (10)$$

100 where $\omega(t, j)$ is the supply of permits by coalition j and $p(t)$ is the permit price on carbon market, at period t .

2.4. Production functions

The CES production functions are introduced in the following constraints (for coalition j and period t):

105 *General economic good production*

$$Y(t, j) - A_0(j)tg(t, j) \left[\alpha_{0K}K_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0L}L_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0E}E_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} \right]^{\frac{s_0(j)}{s_0(j)-1}} \leq 0. \quad (11)$$

Negative emissions production

$$v(t, j) - A_3(j)tgv(t, j) \left[\alpha_{3K}(j)K_3(t, j)^{\frac{s_3(j)-1}{s_3(j)}} + \alpha_{3L}(j)L_3(t, j)^{\frac{s_3(j)-1}{s_3(j)}} + \alpha_{3E}(j)E_3(t, j)^{\frac{s_3(j)-1}{s_3(j)}} \right]^{\frac{s_3(j)}{s_3(j)-1}} \leq 0. \quad (12)$$

Labour use

$$L(t, j) \geq L_0(t, j) + L_3(t, j). \quad (13)$$

Bounds on sequestration

The negative emissions $v(t, j)$ must be stored. The sequestration potential $BCCS(j)$ can be limited in the different regions of the world.

$$v(t, j) \leq BCCS(j). \quad (14)$$

Useful energy production

$$E_0(t, j) + E_3(t, j) - A_e(j) \left[\alpha_{Ef}(j)enf(t, j)^{\frac{s_e(j)-1}{s_e(j)}} + \alpha_{Er}(j)enr(t, j)^{\frac{s_e(j)-1}{s_e(j)}} \right]^{\frac{s_e(j)}{s_e(j)-1}} \leq 0. \quad (15)$$

110 *Fossil secondary energy production*

$$enf(t, j) - A_1(j) \left[\alpha_{1K}(j)(tgenf(t, j)K_1(t, j))^{\frac{s_1(j)-1}{s_1(j)}} + \alpha_{1em}(j)enf_1(t, j)^{\frac{s_1(j)-1}{s_1(j)}} \right]^{\frac{s_1(j)}{s_1(j)-1}} \leq 0. \quad (16)$$

Renewable secondary energy production

$$enr(t, j) - A_2(j)(tgenr(t, j)K_2(t, j))^{s_2(j)} \leq 0. \quad (17)$$

The elasticity (s .) and share parameters (α .), obtained from calibration are shown in Table 12. The parameters $tg(t, j)$, $tgv(t, j)$, $tgenf(t, j)$, $tgenr(t, j)$ are exogenously defined productivity growth factors.

115 *2.5. Carbon market equilibrium*

The constraints describing the international carbon market are given below. The strategic variable, for each coalition j , is the quantity of emission rights $\omega(t, j)$ they supply to the market at period t . On the carbon market the total supply of permits must be greater or equal to total emissions. The firms, in
 120 each coalition, will set their emission at a level where carbon price equals the marginal productivity of emissions (or marginal abatement cost). These two sets of conditions determine the market equilibrium:

Emissions from primary fossil energy (for coalition j at period t)

$$em(t, j) = Coeff(j) \times enf_1(t, j), \quad (18)$$

where the emission rate is evaluated at $Coeff(j) = 0.004$ GtCO₂ per PJ of
 125 fossil energy source.

Total supply of permits is greater or equal to total emissions (at period t)

$$\sum_j \omega(t, j) - \sum_j em(t, j) \geq 0. \quad (19)$$

Efficiency (at period t)

$$p(t) = \frac{\partial Y(t, j)}{\partial em(t, j)} \quad (20)$$

$$= \frac{\partial Y(t, j)}{\partial E_0(t, j)} \frac{\partial E_0(t, j)}{\partial enp_1(t, j)} \frac{\partial enp_1(t, j)}{\partial em(t, j)}. \quad (21)$$

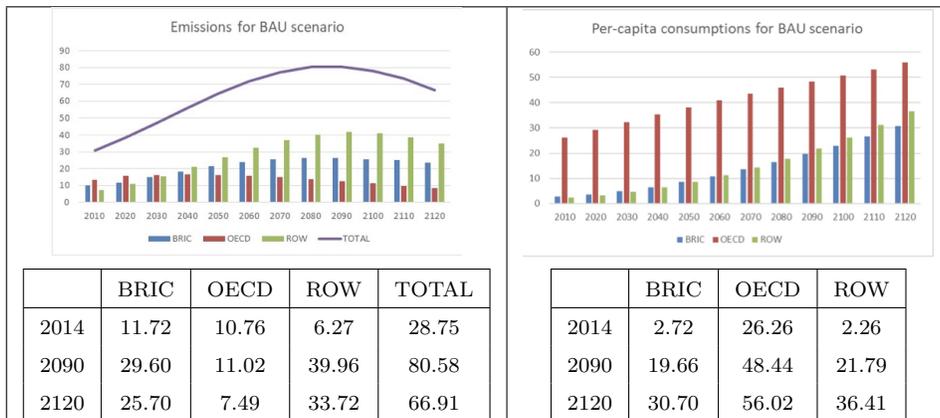
The expression of the derivatives are given in Appendix.

3. Scenarios

3.1. BAU optimal growth

In this simulation the sum for the three regions of the discounted utilities derived from consumption is optimized over a 150 year horizon. To eliminate the end-of-horizon effects we look only at the variable values between 2014 and 2120. In the BAU⁹ scenario, there is no possibility to use CDR and one assumes a very large SEB, i.e. $B = 100'000\text{Gt}$. This eliminates the emissions constraints for all practical purpose. The resulting global and regional emission profiles that are associated with economic growth are shown in Table 1. Yearly emissions top at 80 Gt CO₂ in 2090 and decline afterwards. This is due to the increase in efficiency of renewable energy technologies. Cumulative emissions on the next 150 years reach more than 9'170 Gt CO₂. This is 8 times the nominal SEB of 1'170 Gt CO₂. The temperature rise and the resulting damages would be considerable. Per-capita consumption more than double for OECD countries, from 2020 to 2130, but it is multiplied by a factor 11 for BRIC and a factor 16 for ROW.

Table 1: BAU scenario: Emission and consumption profiles



⁹Business as usual.

145 The considerable increase in fossil energy and renewable energy capital stocks is summarised in Table 2.

Table 2: BAU scenario: K_1/L versus K_2/L

K_1/L				K_2/L			
Year	BRIC	OECD	ROW	Year	BRIC	OECD	ROW
2014	1.56	3.82	0.89	2014	2.17	7.90	2.32
2120	21.52	28.94	22.00	2120	36.98	52.76	65.83

3.2. GREEN: Convergence to ZNE without CDR/DAC

The global SEB¹⁰ of 1'170 GtCO₂ is used, cooperatively, in a program where the sum for the three regions of the discounted utilities derived from consumption is optimized over a 150 year horizon. By setting to 0 the upper limit on carbon sequestration we eliminate the possibility to use CDR/DAC technologies. The only option to reduce emissions is to switch to renewable energy source. The resulting global and regional emission profiles that are associated with economic growth are shown in Table 3. From 2020 to 2130, per-capita consumption is multiplied by a factor 1.8 for OECD countries, but it is multiplied by a factor 8 for BRIC and a factor 13 for ROW.

The fossil energy capital stock collapses and renewable energy capital stock reaches much higher value as shown in Table 4.

¹⁰For a justification of this SEB we refer to the recent IPCC report [17]

Table 3: GREEN scenario: Emission and consumption profiles

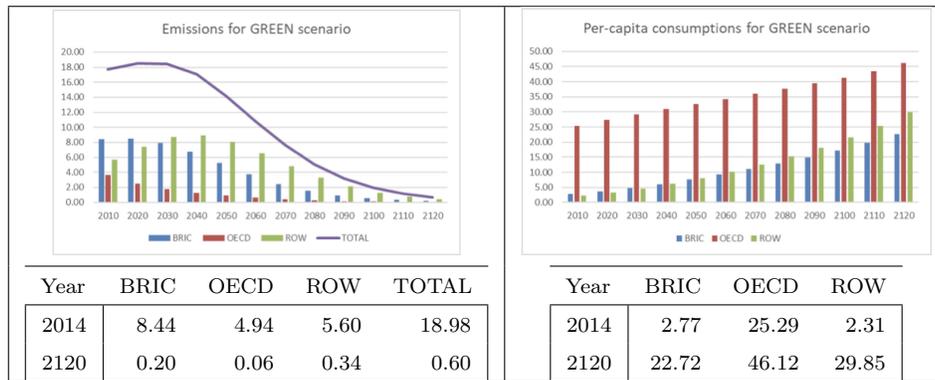


Table 4: GREEN scenario: K_1/L versus K_2/L

K_1/L				K_2/L			
Year	BRIC	OECD	ROW	Year	BRIC	OECD	ROW
2014	1.56	3.82	0.89	2014	2.17	7.90	2.32
2120	0.39	0.55	0.51	2120	50.10	80.25	76.28

3.3. MARKET: Optimal use of shares of SEB with carbon market

160 In this scenario we give the coalitions a share of the SEB and a possibility to capture CO₂ as indicated on Table 5.

Table 5: SEB shares and sequestration bounds

Budget shares $\theta(\cdot)$			DAC-CCS Bounds (Gt CO ₂ /Y)		
BRIC	OECD	ROW	BRIC	OECD	ROW
0.4	0.1	0.5	8	5	10

A zero-net regime is reached by year 2070, as shown on Figure 1. CDR/DAC activity begins in 2060 with a rapid increase until 2090 when it reaches a steady-state at 23 Gt CO₂ captured each year, as shown on Figure 2. The resulting global and regional emission profiles that are associated with economic growth
 165 are shown in Table 6.

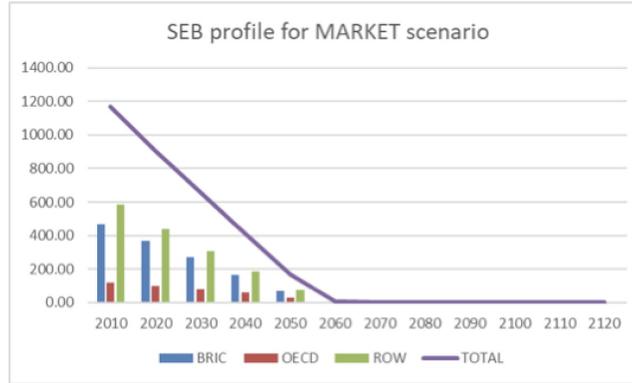


Figure 1: Budget Profiles

From 2020 to 2130, per-capita consumption net of revenue from permit trading is multiplied by a factor 2 for OECD countries, but it is multiplied by a factor 9.6 for BRIC and a factor 14.4 for ROW. The emissions decline until
 170 17 Gt/Y in 2070 and reach a steady state at 21.4 Gt/Y thereafter.

Table 7 gives the per-capita consumption corrected by the revenue or spending associated with emissions trading.

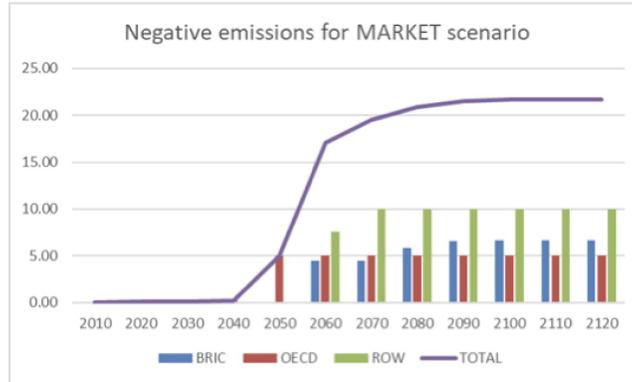


Figure 2: CDR/DAC activity

Table 6: DAC/MARKET scenario: Emission and consumption profiles



Table 7: Optimal consumption net of revenue from permit trading

	BRIC	OECD	ROW
2014	2.77	25.74	2.30
2120	26.60	53.89	33.20

Table 8: MARKET scenario: Carbon price and permits trading

Price (\$)		Trading (per-capita)			Negative emissions (Gt CO ₂)		
Year	BRIC	BRIC	OECD	ROW	BRIC	OECD	ROW
2014	241.30	0.20	-2.03	0.66	0.00	0.00	0.00
2070	761.23	-0.40	1.44	-0.12	4.52	5.00	10.00
2120	828.36	-0.07	1.43	-0.26	6.44	5.00	10.00

Carbon price starts at \$ 241 in 2020 and reaches \$ 828 in 2130 (See Table 8).
 BRIC and ROW are permit sellers and OECD is a permit buyer, as indicated
 175 on Table 8. Table 9 shows the evolution of fossil and renewable capital stocks,
 expressed in per-capita values.

Table 9: MARKET scenario: K_1/L versus K_2/L

K_1/L				K_2/L			
Year	BRIC	OECD	ROW	Year	BRIC	OECD	ROW
2014	1.56	3.82	0.89	2014	2.17	7.90	2.32
2120	8.11	10.87	9.82	2120	39.81	60.88	67.18

3.4. Comparing welfare

We use the discounted sum of per-capita consumption $W(j)$ defined in (10)
 as a welfare criterion. Table 10 shows the welfare losses with respect to the
 180 BAU scenario. From these figures it appears that introducing DAC technologies
 mitigates the welfare losses w.r.t. BAU scenario. We indicate also the welfare
 losses for a scenario OPT where the SEB is used optimally, without using a
 carbon market, in order to optimise the total discounted sum of the utilities
 derived from consumption (we do not give the details of the run due to the lack
 185 of space). It is clear that the MARKET scenario is sub-optimal but very close
 to the optimal one. The variations in the welfare losses seem to indicate that
 giving 10 % of the SEB to OECD is too generous and 50 % of the SEB to ROW
 is not sufficient.

In this analysis we have uniquely considered the economic variables that
 190 are affecting welfare. Indeed, BAU scenario would create considerable damages
 that could not be easily translated into economic losses (e.g. bio-diversity loss
 and species extinction). Also the stranded asset risk, which exists in a GREEN
 scenario where fossil energy disappears almost completely should be considered
 and included in a policy analysis.

Table 10: Welfare criteria

Welfare	MARKET	OPT	BAU	GREEN
BRIC	231.25	231.60	254.40	220.161
OECD	1094.70	1087.24	1156.62	1029.48
ROW	248.06	248.70	262.59	238.551
Welfare Loss	MARKET	OPT	BAU	GREEN
BRIC	9%	9%	0%	13%
OECD	5%	6%	0%	11%
ROW	6%	5%	0%	9%

195 4. Discussion and Conclusion

The objective of this short paper was to present a compact dynamic opti-
 misation model that can give a first insight into the importance of developing
 DAC technologies to cope with anthropogenic climate change. The model is
 implemented in AMPL¹¹. The first scenarios, BAU or GREEN, obtained with
 200 this compact OR model compare well with the scenarios constructed with much
 larger and more encompassing AIMS [18]. The MARKET scenario is relatively
 original as it provides strong support for the consideration of DAC activities as
 a promising way to achieve the objectives of the Paris Agreement in the long
 term.

205 The interest in having developed a compact model lies in the possibility of
 introducing stochastic control and/or dynamic game techniques into the mo-
 delling to address issues of uncertainty and strategic behaviour. Because it is

¹¹The source files can be obtained on request.

small and based on a consistent use of nested CES production functions, this model clarifies the challenges and opportunities of developing climate policy in
 210 a framework of economic development and growth.

5. Appendix

5.1. Figure and Tables

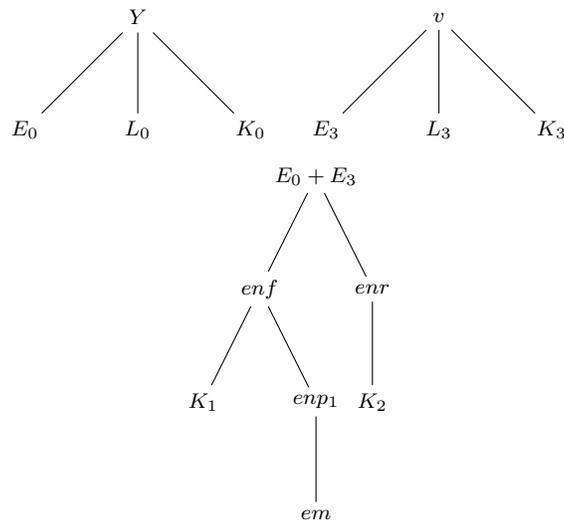


Figure 3: Nested structure for general economy and CDR activity

Table 11: Population levels

Year	2014	2020	2030	2040	2050	2060	2070	2080	2090	2100
BRIC	3042	3251	3375	3417	3387	3303	3182	3043	2904	2822
OECD	1273	1331	1365	1385	1388	1382	1372	1363	1354	1350
ROW	2980	3527	4081	4626	5141	5602	6001	6331	6587	6703
TOTAL	7295	8109	8821	9428	9916	10287	10555	10737	10845	10875

Table 12: CES functions parameters

Y	OECD	BRIC	ROW		v	OECD	BRIC	ROW
$A_0(\cdot)$	6.853	2.329	2.193		$A_3(\cdot)$	0.006	0.002	0.002
$\alpha_{0K}(\cdot)$	0.398	0.298	0.284		$\alpha_{3K}(\cdot)$	0.588	0.556	0.545
$\alpha_{0L}(\cdot)$	0.518	0.454	0.510		$\alpha_{3L}(\cdot)$	0.243	0.286	0.286
$\alpha_{0E}(\cdot)$	0.085	0.248	0.207		$\alpha_{3E}(\cdot)$	0.169	0.158	0.158
$s_0(\cdot)$	0.9	0.9	0.9		$s_3(\cdot)$	0.9	0.9	0.9
$E_0 + E_3$	OECD	BRIC	ROW		enf	OECD	BRIC	ROW
$A_e(\cdot)$	6.853	2.329	2.193		$A_1(\cdot)$	6.853	2.329	2.193
$\alpha_{Ef}(\cdot)$	0.706	0.702	0.616		$\alpha_{1K}(\cdot)$	0.517	0.434	0.693
$\alpha_{Er}(\cdot)$	0.294	0.298	0.384		$\alpha_{1em}(\cdot)$	0.483	0.566	0.307
$s_e(\cdot)$	1.5	1.5	1.5		$s_1(\cdot)$	1.5	1.5	1.5
					enr	OECD	BRIC	ROW
					$A_2(\cdot)$	0.095	0.087	0.114
					$s_2(\cdot)$	1	1	1

5.2. Derivatives of the production function

$$\frac{\partial Y(t, j)}{\partial E_0(t, j)} = A_0(t, j)tg(t, j) \left[\alpha_{0K}K_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0L}L_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} + \alpha_{0E}E_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}} \right]^{\frac{s_0(j)}{s_0(j)-1}-1} \alpha_{0E}E_0(t, j)^{\frac{s_0(j)-1}{s_0(j)}-1}, \quad (22)$$

$$\begin{aligned} \frac{\partial E_0(t, j)}{\partial enr_1(t, j)} &= \frac{\partial E_0(t, j)}{\partial enf(t, j)} \frac{\partial enf(t, j)}{\partial enr_1(t, j)} = \\ &= \frac{A_e(j)}{Coeff(j)} \left[\alpha_{Ef}(j)enf(t, j)^{\frac{s_e(j)-1}{s_e(j)}} + \alpha_{Er}(j)enr(t, j)^{\frac{s_e(j)-1}{s_e(j)}} \right]^{\frac{s_e(j)}{s_e(j)-1}-1} \\ &= \alpha_{Ef}(j)enf(t, j)^{\frac{s_e(j)-1}{s_e(j)}-1} A_2 \left[\alpha_{1K}(j)(enf(t, j)K_1(t, j))^{\frac{s_1(j)-1}{s_1(j)}} + \alpha_{1em}(j)(j)enf_1(t, j)^{\frac{s_1(j)-1}{s_1(j)}} \right]^{\frac{s_1(j)}{s_1(j)-1}-1} \\ &= \alpha_{1em}(j)(j)enf_1(t, j)^{\frac{s_1(j)-1}{s_1(j)}-1} \end{aligned} \quad (23)$$

$$\frac{\partial enp_1(t, j)}{\partial em(t, j)} = \frac{1}{Coeff(j)}.$$

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